

## Seminar im Rahmen des GRK 2078

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Date: Tuesday, February 5, 2019  
Time: 14:00 h  
Location: Bldg. 10.23, 3rd Floor, Room 308.1 (KM-Seminar Room)

Title: **New advancements on the J-, M-, and L-integrals:  
from micromechanics of dislocations to body charges and body forces**

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### Abstract

In this work, a new insight is given into the **J**-, **M**-, and **L**-integrals and their physical interpretation. We start with the **J**-, **M**-, and **L**-integrals of straight dislocations which are derived in the framework of three-dimensional, incompatible, linear elasticity [1]. The **J**-integral of dislocations is the well-known Peach-Koehler force (interaction force) between the two dislocations. The obtained results reveal the physical interpretation and significance of the **M**-, and **L**-integrals for straight dislocations. In particular, it is shown that the **M**-integral between two straight dislocations (per unit dislocation length) is half the corresponding interaction energy between the two dislocations (per unit length) plus twice the corresponding pre-logarithmic energy factor. This result gives for the first time to the **M**-integral the physical interpretation of the interaction energy (depending on the distance and on the angle) between the two straight dislocations. The  $L_z$ -integral of two straight dislocations is the z-component of the configurational vector moment or the rotational moment (torque) about the z-axis caused by the interaction of the two dislocations. Moreover, the **J**-, and  $L_z$ -integrals are interpreted as translational and rotational energy-release, respectively. The chosen framework of three-dimensional, incompatible (consideration of plastic fields) elasticity is able to capture the interaction between the two dislocations.

Next, the **J**-, **M**-, and **L**-integrals of a single dislocation (edge and screw) are derived as a limit of the **J**-, **M**-, and **L**-integrals between two straight dislocations in isotropic elasticity [2]. The remarkable outcome is that the **M**-integral (per unit length) represents the total energy (per unit length) of the dislocation which is given by the sum of the self-energy (per unit length) and the dislocation core energy (per unit length). The latter can be identified with the configurational work produced by the Peach-Koehler force. It is shown that the dislocation core energy (per unit length) is twice the corresponding pre-logarithmic energy factor. This result is valid in isotropic as well as in anisotropic elasticity. The only difference lies on the pre-logarithmic energy factor which is more complex in anisotropic elasticity due to the anisotropic energy coefficient tensor which captures the anisotropy of the material.

In the end, the **J**-, **M**-, and **L**-integrals of body charges and point charges in electrostatics, and the **J**-, **M**-, and **L**-integrals of body forces and point forces in elasticity are presented and their physical interpretation is investigated [3]. One of the basic quantities appearing in the **J**-, **M**-, and **L**-integrals is the electrostatic Maxwell-Minkowski stress tensor in electrostatics and the Eshelby stress tensor in elasticity. Among others it is shown that the **J**-integral of body charges in electrostatics represents the electrostatic part of the Lorentz force, and the **J**-integral of body forces in elasticity represents the Cherepanov force. The **M**-integral between two point sources (charges or forces) equals half the electrostatic interaction energy in electrostatics and half the elastic interaction energy in elasticity between these two point sources. The **L**-integral represents the configurational vector moment or torque between two body or point sources (charges or forces). Interesting mathematical and physical features are revealed through the connection of the **J**-, **M**-, and **L**-integrals with their corresponding infinitesimal generators in both theories. Several important outcomes arise from the comparison between the examined concepts in electrostatics and elasticity. Differences and similarities, that provide a deeper insight into the **J**-, **M**-, and **L**-integrals and the related quantities to them, are pointed out and discussed. The presented results show that the **J**-, **M**-, and **L**-integrals are fundamental concepts which can be applied in any field theory.

## References

- [1] E. Agiasofitou and M. Lazar, Micromechanics of dislocations in solids: **J**-, **M**-, and **L**-integrals and their fundamental relations, *Int. J. Eng. Sci.* **114**, 16-40, 2017.
- [2] M. Lazar and E. Agiasofitou, Eshelbian dislocation mechanics: **J**-, **M**-, and **L**-integrals of straight dislocations, Special Issue G.A. Maugin, *Mech. Res. Commun.* **93**, 89-95, 2018.
- [3] M. Lazar and E. Agiasofitou, The **J**-, **M**-, and **L**-integrals of body charges and body forces: Maxwell meets Eshelby, *Journal of Micromechanics and Molecular Physics*, (34 pages), doi: <https://doi.org/10.1142/S242491301840012X>, 2018.

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Alle Interessenten sind herzlich eingeladen.

Prof. Dr.-Ing. Thomas Böhlke  
(Sprecher des GRK 2078)